Mixing and clustering in compressible chaotic stirred flows

Vicente Pérez-Muñuzuri*
Group of Nonlinear Physics, Faculty of Physics, University of Santiago de Compostela. E-15782 Santiago de Compostela, Spain

Transport of inertial (finite-size) particles in flows shows properties typical of compressible fluids, even in incompressible flows. The most unexpected behavior is the formation of clusters of particles out of an initially homogeneous distribution. In the limit the Stokes drag is very strong, inertial particles recover the motion of Lagrangian tracers and no clustering should be expected[1]. However, passive tracers moving on the surface of an incompressible flow may lead to the formation of cluster structures[2].

The effect of compressibility on the mixing of Lagrangian tracers is analyzed in chaotic stirred flows. Mixing is studied in terms of the Finite-Time Lyapunov Exponents (FTLE)[3]. Mixing and clustering of passive tracers surrounded by Lagrangian coherent structures is observed to increase with compressibility intensity. The role of the stirring rate and compressibility on mixing and clustering has been analyzed.

We investigate the effects of compressibility in the periodically varying double-gyre flow[4,5],

$$u = -\pi A \frac{\rho_0}{\rho} \sin(\pi f(x,t)) \cos(\pi y)$$
$$v = \pi A \frac{\rho_0}{\rho} \frac{\partial f}{\partial x} \cos(\pi f(x,t)) \sin(\pi y), \tag{1}$$

where

$$f(x,t) = a(t)x^2 + b(t)x, \tag{2}$$

over the domain $[0,2] \times [0,1]$. $a(t) = u_0 \sin \omega t$ and $b(t) = 1 - 2u_0 \sin \omega t$. $T = 2\pi/\omega$ and A are the period and amplitude of the flow, respectively. The periodic perturbation leads to mixing between the two gyres.

In order to satisfy the continuity equation, $\partial_i(\rho u_i) = 0$, the spatial dependence of the flow density can be written as,

$$\rho(x,y) = \rho_0 \left[1 + \epsilon \sin(2\pi x/\lambda) \sin(2\pi y/\lambda)\right], \tag{3}$$

where ϵ and λ are the compressibility of the flow and wavelength, respectively.

We analyze the effects of ϵ and λ on the mixing of the double-gyre flow in terms of the FTLE fields. Compressibility perturbation wrinkles the LCS in the small-wavelength limit, whereas for the large wavelengths LCS are slightly distorted since the entire domain is embedded in nearly a wavelength (Fig. 1). The density profile favors some initial clustering of the particles that is broken by the periodically contracting and expanding of the gyres. Then particles move within the interstices between LCS.

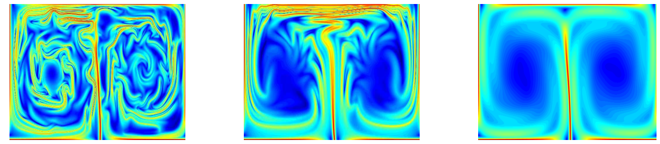

FIG. 1. Finite-Time Lyapunov Exponent σ for the double-gyre flow. $\lambda = 0.4$ (left panel) and $\lambda = 0.8$ (central panel). Right panel shows the FTLE for the incompressible flow model for comparison ($\epsilon = 0$).

Particles belonging to these clusters are characterized by a negative value of the FTLE, while for an incompressible flow ($\epsilon = 0$) the FTLE values are always positive. For any wavelength λ of the density field, as ϵ increases, mixing and clustering are enhanced. The mean FTLE and the variability of the FTLE increase with the compressibility. The increase of variability is due to an increase in the number of tracers with negative FTLE followed by an increase in the number of clusters or aggregates of these particles. Both, the mean and the variability of the FTLE, decrease with increasing λ. However, while the number of tracers with negative Lyapunov exponent hardly it changes with λ, the number of clusters and their size diminish with increasing λ. In other words, at fixed ϵ, clusters are smaller but more populated at large values of λ.

In the considered flow, mixing is strongly affected by compressibility, and the compressibility field forces a strong localization of density[?,2,6]. Formation of clusters separated by Lagrangian coherent structures has been analyzed in terms of ϵ. Cluster formation is enhanced as compressibility increases based on the combination of particles attracted to areas with large compression $\rho/\rho_0 > 1$, and detaching of patches of particles from these initial clusters that wander among the chaotic flow. For enough large stirring rate the flow is quenched and clusters survive forever for any wave length and compressibility intensity.

* vicente.perez@cesga.es

[1] J. Bec, Phys. Fluids **15**, L81–L84 (2003); G. Boffeta, F. de Lillo, and A. Gamba, Phys. Fluids **16**, L20–L23 (2004).
[2] V.I. Klyatskin and A.I. Saichev, J. Exp. Theor. Phys. **84**, 716–724 (1997).
[3] Z. Neufeld and E. Hernández-García, *Chemical and biological processes in fluid flows* (Imperial College Press, 2009).
[4] S.C. Shadden, F. Lekien, and J. Marsden, Physica D **212**, 271–304 (2005).
[5] V. Pérez-Muñuzuri and F. Huhn, Nonlin. Processes Geophys. **20**, 987–991 (2013).
[6] F. Bianco, S. Chibbaro, D. Vergni and A. Vulpiani, Phys. Rev. E. **87**, 042924 (2013).