Exact results in two-dimensional domain growth

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Dynamical systems quenched from a disorder into an ordered phase, may display coarsening phenomena. The simplest example is the Ising ferromagnet. When the system is cooled rapidly through the transition temperature, domains of the two ordered phases form and grow (coarsen) with time.

A common feature of coarsening systems is the dynamical scaling hypothesis: the domain morphology is statistically the same at all times when all lengths are measured in units of a characateristic length scale $L(t)$. Despite the success of the scaling hypothesis in describing experimental and simulation data, its validity has only been proved for very simple models, including the exactly soluble one-dimensional Glauber-Ising model and the nonconserved $O(n)$ model in the limit $n >> 1$.

In the present work we obtain the exact result for the statistics of the areas enclosed by domain boundaries (hulls) for the coarsening dynamics of a nonconserved scalar field in two dimensions, demostrating explicity, en passant, the validity of the scaling hypothesis for this system.

Using the Allen-Cahn equation and an elegant application of the Gauss-Bonnet theorem, we show[1,2] that the number of hulls per unit area that enclose an area greater than A, in a system evolving at zero temperature from a disordered initial state, has for long time t the form $N_h(A,t) = c/(A + \lambda t)$ where $c = 1/8\pi\sqrt{3}$ is a universal constant introduced by Cardy and Ziff in the context of percolation theory. The same form is obtained for coarsening from a critical initial state, but with c repaced by $c/2$.

Notice that our solution has the expected scaling form corresponding to a system with characteristic length scale $L(t) \propto t^{1/2}$, which is the known result if scaling is assumed. Here, however, we do not assume scaling, rather, it emerges from the calculation.

We also prove that the domain area distribition (where domains are the areas of aligned spins), are almost identical to the hull distribution. These results can be generalizated to the coarsening dynamics under the effect of finite temperature[3] or the presence of quenched disorder[4] in the system. The full temperature or disorder dependence enters only through the value of then, characteristic length scale $L(t)$.

Our analytical results have been tested with simulations on the two-dimensional square-lattice Ising model using a Montecarlo algorithm. Numerical data are in excellent agreement with predictions.

An experimental test is also being done. A setup has been developed by professor's Dierking group at Manchester in a liquid crystal system that, under an applied electric field, spontaneously separates into phases with left and right-handed chirality. The experiments are still in course, results will be available in some weeks.

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