Perfect Plasticity and Shear Deformation in a Random Medium

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Many materials in nature exhibit elasto-plastic behavior. Under small load, they show an elastic regime in which the strain is proportional to the applied stress and deformations are reversible. Beyond the yield point, they display plastic behavior and deformations are irreversible. Perfect plasticity is characterized by the fact that the strain can increase indefinitely without further increase of the stress. Most commonly, what is observed is "hardening", i.e., the stress required to deform the material increases with the strain. For crystalline materials, plasticity can be explained by the motion and interactions among dislocations of crystalline planes but for amorphous media the origin of plastic behavior is still unclear.

Here we study the behavior of a random medium subject to an increasing external stress within the framework of the random fuse model (RFM), electrical analog of the elasticity problem. One can define a plastic version of the RFM[1] letting the fuses behave linearly with a conductivity g until the current (stress) reaches a threshold value i_{th} and then keeping it constant even though the voltage (strain) continues to increase. We perform numerical simulations to study the macroscopic mechanical response of the model and the scaling with system size as we approach the plastic steady state. We average the results over a large number of realizations and find that the yield stress distribution obeys Gaussian statistics. We will also address here the common belief that this problem belongs to the random bond Ising model universality class since the yield current of the system is given by the sum of the individual currents along the minimum current interface. In this model there is no local rearrangement of stress each time a bond plastifies, hence the strain avalanches, commonly observed in experiments are not reproduced.

We thus consider a more realistic model, following the spirit of models developed for tectonic plates[2] where a local slip event occurs when a bond reaches its threshold. Then, an irreversible strain is imposed on the bond, leading to a decrease in its elastic stress. In the RFM, this process can be simulated by adding a voltage source $V_s = \beta i_{th}/g$ to the element in a way to generate an opposite current in the bond. After that, the bond is healed and becomes elastic again. The consequent redistribution of stress throughout the system can cause more slip events, leading to avalanche dynamics. We observe a temporal and spatial localization of the plastic events into several directed paths coincident with minimal energy paths of the system. In the limit of infinitesimal β, this model is equivalent to the perfectly-plastic one but its versatility allows us to study also the effect of hardening and the influence of strain accumulation on the final fracture of the material.

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