Phase transition in a stochastic prime number generator

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Prime numbers are mostly found using the classical sieve of Eratosthenes and its recent improvements . Additionally, several methods able to generate probable primes have been put forward . In this contribution we study an algorithm somewhat different from those mentioned above, based on an artificial chemistry model that generates primes by means of a stochastic integer decomposition.

Suppose a pool of positive integers $\{2, 3, ..., M\}$, from which N numbers are randomly extracted. Notice that the number 1 is ignored and that repetitions are allowed. Given two integers a and b (taken from the set of N numbers), the reaction rules of the algorithm are:

- Rule 1: If $a = b$ then no reaction takes place, and the numbers are not modified.
- Rule 2: If the numbers are different, say $a > b$, a reaction will take place only if b is a divisor of a, i.e. if there exists an integer $c = a/b$. Then a is eliminated from the set and substituted by c.
- Rule 3: On the other hand, if a is not divisible by b, then no reaction takes place.

The stochastic algorithm goes as follows: after choosing N numbers from the pool $\{2, 3, ..., M\}$ two numbers a,$b \in N$ are picked at random; then the reaction rules are applied to them. We consider N repetitions of this process as one time step in order to have a parallel updating. Notice that the positive reactions tend to decompose numbers, thereby this process when iterated may generate primes. We say that the system has reached the steady state when no more reactions are achieved, either because every number has become a prime or because rule 2 cannot be satisfied anymore: the algorithm then stops.

We will firstly present the phase transition that the system seems to exhibit, between a regime with low prime density and a regime where every number ends as a prime. In a second part we will try to interpret this phase transition in terms of a dynamical process embedded in a directed catalytic network, introducing subsequently a proper order parameter. Some analytical arguments in terms of an annealed approximation will then be outlined in a third part. Finally, some links with computational complexity theory will be addressed.

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[1] Phase transition in a stochastic prime number generator, B. Luque, L.Lacasa and O. Miramontes *Physical Review E* **76**, 010103 (R) (2007)

[2] Phase transition and computational complexity in a stochastic prime number generator, L.Lacasa, B. Luque and O. Miramontes - Submitted to *New Journal of Physics*