Constructive Chaos in Excitable Networks with Tuneable Topologies

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We study an excitable network with tuneable power–law wiring connectivity in which the connection weights vary rapidly with the system activity in a way that mimics resistance, fatigue or synaptic depression. The resulting dynamics has attractors, corresponding to patterns of activity, which are destabilised by the rapid fluctuations, and three main phases emerge: a "ferromagnetic" or memory phase, a phase of chaotic hopping among the patterns, and a phase of periodic pattern–antipattern switching[1]. In a mean–field approach for a single pattern, the dynamics of the network is well approximated by a two–dimensional discrete map. In the case of a random scale–free graph, the exponent of the power–law degree distribution can be used as a control parameter. Analysis of the map then reveals that there is an optimal exponent, around 2, which minimises the amount of excitation (e.g., fatigue) needed to destabilise the memory patterns.

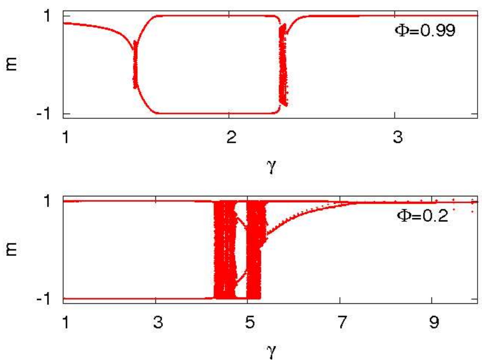

Figura 1. γ dependence for bifurcation diagrams for two different values of depression, $\Phi = 0.99$ and $\Phi = 0.2$, and $\langle k \rangle = 20$, $N = 1600$, $T = 1/800$. Very slight depression ($\Phi \lesssim 1$) is enough to destabilise the patterns around $\gamma \simeq 2$, whereas a stronger depression ($\Phi \to 0$) is required to take the chaotic region to topologies with steeper distributions – i.e. with a higher power-law exponent.

Our study shows that certain topologies are most convenient for storage and retrieval of memories, whereas others allow for a better performance in tasks requiring unstable behaviour, such as pattern recognition and class identification and categorisation. We go on to suggest a mechanism by which a network could switch from stable to unstable behaviour by means of only a slight rewiring. Our Monte Carlo simulations agree both qualitatively and quantitatively with the mean–field results.

Though possibly a general feature of excitable media with fatigue, we believe these results to be particularly relevant for neural networks, since recent evidence suggests the existence of chaotic brain activity[2] and the emergence of a scale-free functional topology – with an exponent close to 2 – during cognitive processes[3].

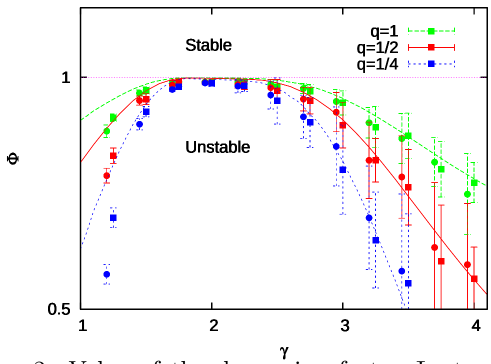

Figura 2. Value of the depression factor Φ at which the behaviour of m becomes chaotic, Φ_{chaos}, for depression probabilities $q = 1, 1/2$ and $1/4$, as obtained from the map (lines) compared to MC simulations for one pattern (squares) and two (circles). Bars represent standard deviation. (Data correspond to averages over 10 network realisations, with $\langle k \rangle = 20$, $N = 1600$, and $T = 1/800$.)

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