Dynamical instabilities of dissipative solitons in nonlinear optical cavities with nonlocal metamaterials

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In this work we characterize the dynamical instabilities of Localized Structures (LS) exhibited by a recently introduced generalization[1] of Lugiato-Lefever model[2] including the weakly *nonlocal* response of an intracavity metamaterial:

$$\frac{\partial E}{\partial t} = -(1+i\theta)E + E_{in} + i|E|^2\,E + i\nabla_\perp^2 E + i\beta\nabla_\perp^4 E. \tag{1}$$

This equation describes the temporal evolution of the slowly varying envelope of the electric field $E(\vec{x},t)$ in a double-layered optical cavity. One layer of the cavity consists of a conventional right-handed material, while the other layer is an optical left-handed metamaterial. The bilaplacian term accounts for a linear weakly nonlocal response of the left-handed metamaterial. Furthermore, it has been shown that in this double-layered cavity β can be drastically altered by changing the relative lengths of both material layers.

We observe a rich scenario (See Fig.1), in which the LS exhibit different types of oscillatory instabilities, and excitability. We shown that the scenario is organized by a pair of Takens-Bogdanov (TB) codimension-2 points. The dynamical regimes arising from the TB_1 are qualitatively the same as observed without intracavity metamaterials[3], but now the TB point, located at the conservative limit ($\theta \to \infty$) in the previous case, is located at finite parameter values as a result of the nonlocality.

The TB_2 leads, however, to completely new dynamical regimes. Particularly interesting is a regime of conditional excitability where the system is simultaneously excitable and bistable. In this regime, perturbations that are not able to cross the excitability threshold lead to normal relaxation to the fundamental state. Perturbations of moderate intensity above the excitability threshold lead to a long excursion in the phase space, while perturbation of higher intensity, above a second threshold, will form a stable high amplitude LS. So the dynamical response to perturbations is more complex than simply sub- and supra-threshold, and for the latter type of perturbations two possible regimes are possible.

Finally we will discuss how this scenario might emerge from the collapse dynamics of solitons in the two dimensional Nonlinear Schrödinger Equation.

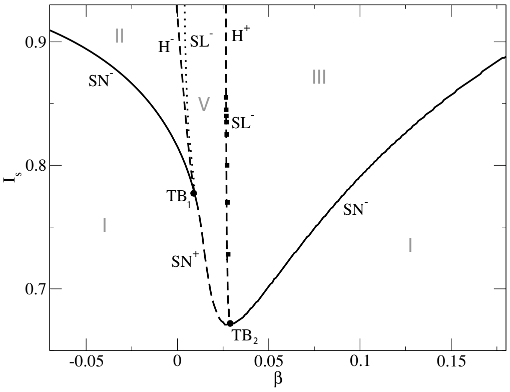

Figura 1. Top: Phase diagram of localized structures in a Kerr cavity with metamaterials. LS are stable between the Saddle-Node bifurcation (solid line - SN) and the Hopf bifurcation (dashed line - H), and oscillate between the Hopf bifurcation and the Saddle-Loop (homoclinic) bifurcation (dotted line - SL). Beyond the homoclinic bifurcation, the system exhibits excitability. Below the saddle-node bifurcation, there exist no LS. The saddle-node and Hopf bifurcation lines meet at two codimension-2 Takens-Bogdanov points. Here $\theta = 1.23$.

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[4] `http://www.ifisc.uib.es`