Information and performance in a feedback controlled Brownian ratchet

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Thermal ratchets or Brownian motors can be viewed as controllers that act on stochastic systems with the aim of inducing directed motion through the rectification of fluctuations[1]. In most cases, the system to be controlled is modelled as a collection of Brownian particles undergoing Langevin dynamics, and the control action—that is, the rectification mechanism—is implemented by applying random or deterministic time-dependent perturbations to the particles. In this context, one can distinguish two types of ratchets: (i) *open-loop* ratchets, which are ratchets that apply a rectifying potential independently of the state of the system to be controlled[1]; (ii) *closed-loop* or *feedback* ratchets, whose rectification action on a system has an explicit dependence on the state of the system.[2]

In this work we establish a quantitative comparison of these two types of ratchets that explicitly focuses on what distinguishes them, namely the use of information. This is done in three steps using a feedback ratchet that performs an instant maximization of the flux[2] as a case example. First, we show how the information used by the system can be quantified using techniques of information theory[3]. Then we study how the performance of that ratchet, measured by the magnitudes of the flux of particles[4] and the maximum power output[5], varies as a function of the amount of information used in the ratchet effect. Finally, the results are compared with those obtained with the open-loop version of the flashing ratchet, which operates without information.

We get analytical expressions for the flux and the maximum power output in one-particle and few-particle feedback ratchets. In addition, we show that the maximum flux that can be attained by changing from a closed-loop to an open-loop ratchet has an upper bound proportional to the square-root of the information, while the maximum power output has an upper bound proportional to the information.

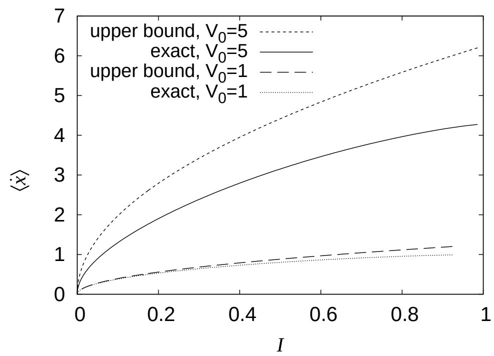

Figura 1. One-particle flux and upper bound vs information for different heights of the ratchet potential.

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