Dynamics of Tidal Synchronization and Orbit Circularization of Celestial Bodies

Bruno Escribano*, Jozsef Vanyo, Idan Tuval, Julyan Cartwright, Diego L. González, Oreste Piro and Tamás Tél
Instituto Andaluz de Ciencias de la Tierra, CSIC
Facultad de Ciencias, Universidad de Granada, Campus Fuentenueva s/n, 18002 Granada

What is the dynamical origin of the fact that the Moon presents the same hemisphere facing perpetually towards the Earth? The other large moons of the solar system also have their rotations synchronized with their orbits, and Pluto and Charon are mutually locked in this way. All of these celestial bodies are in 1:1 spin–orbit resonance. The rotation of one planet, Mercury, is also synchronized with its orbit around the Sun, but it performs three rotations every two orbits, and thus, unlike the former instances, is locked in 3:2 resonance[1]. Similar synchronization phenomena are thought to occur too in solar systems with so-called 'hot Jupiters' or short-period planets[2], and in systems of binary stars[3], whose orbits also evolve to become circular. All these instances are clearly a consequence of a spin–orbit interaction brought about by the gravitational torque exerted by the larger primary body on the smaller secondary body elastically deformed by the differential gravity combined with the corresponding tidal friction induced in the secondary. The phenomenon has long been studied[4,5], but existing models[6–8] are designed for quantitative analysis of a specific instance or a particular part of the problem, and are correspondingly complicated; the details obscure the basic mathematical structure of the dynamical system.

Here we take the opposite course: we study the simplest possible system that displays tidal synchronization and orbit circularization with a minimal model that takes into account only the essential ingredients of tidal deformation and dissipation in the secondary body. In our qualitative dynamical-systems approach, without including the full panoply of details, we treat in a self-consistent way the temporal evolution of the eccentricity and the energy flow from orbital to rotational motion; important ingredients to understand the long-term evolution of the orbit. Despite its simplicity, our model can account for both synchronization into the 1:1 spin-orbit resonance and the circularization of the orbit as the only true asymptotic attractors, together with the existence of relatively long-lived metastable orbits with the secondary in p:q synchronous rotation.

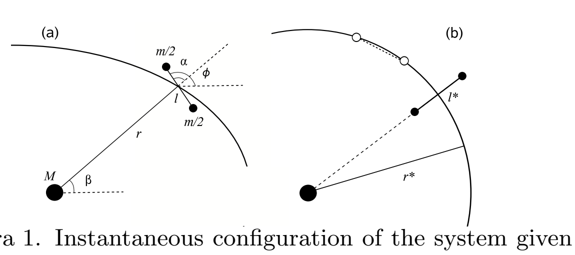

Figura 1. Instantaneous configuration of the system given by the generalized coordinates r, β, l, ϕ. The relative angle $\alpha = \phi - \beta$ is indicated.

We model an extended secondary body of mass m by two point masses of mass $m/2$ connected with a damped spring. This composite body moves in the gravitational field of a primary of mass $M \gg m$ located at the origin. In this simplest case oscillation and rotation of the secondary are assumed to take place in the plane of the Keplerian orbit. We use polar coordinates r, β for the center of mass of the secondary, with l as the instantaneous length of the spring and ϕ the rotational angle characterizing the orientation of the secondary. Both angles β and ϕ are measured from the x-axis in an inertial reference frame. The spring is characterized by its spring constant D and rest length L_0. The gravitational interactions of both point masses with the primary are taken into account, but that between the point masses is neglected.

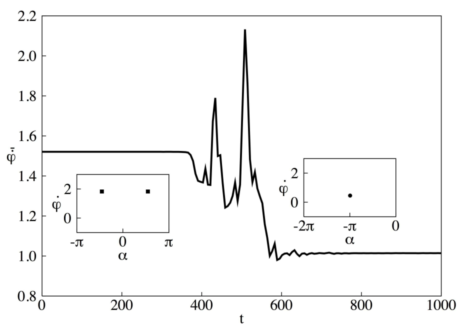

Figura 2. Average angular velocity $\overline{\dot{\phi}}$ versus time for $\varepsilon_0 = 0.2$, $\gamma = \omega = 10$, $l_0 = 10^{-4}$ shows the crossover from 3:2 to 1:1 resonance. The insets show the metastable 3:2 and asymptotic 1:1 attractors on the Poincaré map $\dot{\phi}, \alpha$ taken at apapsis. Time is measured in units of T. The transition shown occurs after the system has spent a long time in the 3:2 resonance and is very abrupt, lasting about 50 periods.

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* bruno@lec.csic.es
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