# Emergence of chaos and criticality in a Neural Network with time dependent connections

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Many experimental works on brain or neural cell culture have demonstrate the existence of fast time variation in connection strength between neurons, the so called short-term synaptic plasticity (for instance, synaptic depression and facilitation). This has been reported to play an important role in memory storage, retrieving, transmission of information and its processing to perform a particular task. Here we present a model, built on the classic neural network model, that considers the effect of short-term synaptic plasticity in the dynamic of the network. In addition, tuning the fraction of neurons $\rho$ which are updated at each time (which mimics different levels of synchronization in the network or the existence of silent neurons) induces the emergence of different behaviours as the standard recovering of memories, regular oscillations between up and down activity states, or chaotic jumping between memories or between oscillating signals[1]. Here, we are interested in the numerical analisis of the chaotic regimes, and we found that a critical state emerges with some statistical observables presenting a typical power law behaviour[3]. Our starting point is a stochastic Hopfield Neural Network with hebbian learning rule to set the synapses which allows for associative memory . We complemented this model with two new dynamical rules which mimic *short-term synaptic plasticity*[2]:

*(i) Time dependent connections.* Consider a network of $N$ binary neurons with states $\sigma_i \in \{-1, 1\}$ (firing or silent), and receiving a synaptic current $h_i = \sum_{j=1}^{N} w_{ij} d_j \sigma_j$. ere, $w_{ij} = \frac{1}{N}\sum_{\nu=1}^{P} \xi_i^{\nu} \xi_j^{\nu}$ are the classical hebbian weights where the system stores $P$ memory patterns $\left\{\vec{\xi^{\nu}}\right\}, \nu = 1, \ldots, P$, and $d_j$ is a depressing factor which considers the stochastic fast synaptic fluctuations due to the overall activity in the network. We assume that $d_j$ in the stationary regime follows a bimodal probability distribution $P_j(d_j) = \zeta_j(\vec{\sigma})\delta(d_j - \Phi) + (1 - \zeta_j(\vec{\sigma}))\delta(d_j - 1)$, which implies that as long as a presynaptic neuron is firing the synapse strenght becomes weaker. This accounts for synaptic depression, that is, the decreasing of the available neurotransmitters in the synaptic button due to a continuous and fast presynaptic stimulation. We assume a standard mean-field framework so that we have $d_j \approx \langle d_j \rangle_{P_j} = [1 - (1 - \Phi)\zeta(\vec{\sigma})]$.

*(ii) Partial updating of neurons.* Recent studies on real neural media show that in many cases neural activity does not have a uniform distribution, and so in a certain time only a portion of neurons are active[4]. Here, we consider this effect using an *hybrid updating* of neurons, that is, each time only a fraction $\rho$ of the neurons, randomly choosed, will be updated. Once we set a level $\Phi$ of synaptic depression, tuning $\rho$ from 0 (sequential Glaubler dynamics) to 1 (parallel Little dynamics), we observe different regimes. For $\rho$ small the system exhibits associative memory and recovers a given pattern. As $\rho$ increases, however, it emerges some chaotic phases, where the activity switchs between the stored patterns or between the pattern and antipattern states, that is $\left\{\vec{\xi^{\alpha}} \leftrightarrow -\vec{\xi^{\alpha}}, \vec{\xi^{\beta}} \leftrightarrow -\vec{\xi^{\beta}}, ..., \vec{\xi^{\gamma}} \leftrightarrow -\vec{\xi^{\gamma}}\right\}$. An analisis of $h_i$ shows that the chaotic regime can be *critical*. Fig. 1(Left) shows the power spectra of $h_i$ in the critical chaotic window $[\rho^c_{Min}, \rho^c_{Max}]$ presenting a power law behaviour, with an exponent depending on $\rho$. The same behaviour appears in the correlation function $C(\tau) = \langle (h(t+\tau) - h(t))^2 \rangle_t$ (data not shown). We explored the origin of such criticality by defining a time permanence probability function, that measures how long $h_i(t)$ fluctuates in a window $\Delta h$, see for instance Fig. 1(Right), until it jumps off. This observable, with a proper choose of the window size, has also a power law behaviour in the chaotic phase. For small window size as well as outside of the window $[\rho^c_{Min}, \rho^c_{Max}]$, it follows an exponential law which suggests a complex interplay between thermal fluctuations, synaptic fluctuations due to the $h_i$ modulation by synaptic depression, and partial neuron updating, to induce the critical state which is caracterized by irregular jumps of the network activity between memories.

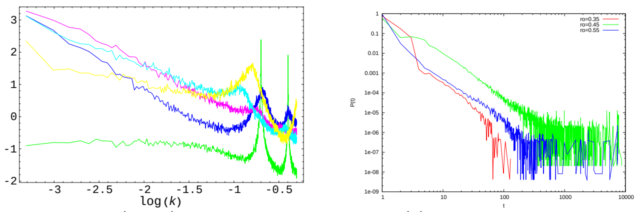


Figura 1. (Left) Power spectra of $h_i(t)$. For wave number $k < -1$ it switchs from constant at $\rho = 0.35$ (in the edge of chaos but still recovering a pattern) to a power law behaviour for $\rho > 0.35$ (chaotic phase). (Right) Jump probability for $h_i(t)$. Note the exponential behaviour (red curve) just below the chaotic region, and the power law behaviour when the system is jumping irregularly between the memory patterns (other curves).

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