Interaction of oscillating dissipative solitons in nonlinear optical cavities

Adrian Jacobo, Damià Gomila, Manuel A. Matías and Pere Colet*
Instituto de Física Intedisciplinar y Sistemas Complejos, IFISC† (CSIC-UIB)
Campus Universitat de les Illes Balears, E-07122 Palma de Mallorca

Dissipative solitons (DS), are spatially localized structures that appear in several nonlinear dissipative media including nonlinear optical cavities where are also known as cavity solitons[1]. DS may develop oscillatory instabilities, so that oscillate in time while remaining stationary in space, like the oscillons (oscillating localized structures) found in a vibrated layer of sand[2]. The occurrence of these oscillons in autonomous systems has been reported in optical[3] and chemical systems[4]. Here we consider the interaction of two of such oscillating structures.

A prototype model in nonlinear optics displaying DS is the one introduced by Lugiato and Lefever for the slowly varying amplitude of the electrical field $E(\vec{x}, t)$ in the transverse plane of an optical cavity filled with a Kerr medium[5]

$$\frac{\partial E}{\partial t} = -(1 + i\theta)E + i\alpha\nabla_{\perp}^{2}E + E_0 + i\,|E|^2\,E. \qquad (1)$$

∇^2 is the transverse Laplacian, E_0 the homogeneous pump and θ the cavity detuning with respect to E_0. The homogeneous steady state is given by $E_s(1 + i(\theta - I_s) = E_0$. The intracavity background intensity $I_s = |E_s|^2$ can be taken as a convenient control parameter. The region of existence of DS has been characterized[3]. Increasing θ or I_s the DS undergoes a supercritical Hopf bifurcation and starts to oscillate autonomously.

When two static DS are placed in the system at arbitrary positions they interact through the tails. Since the tails are oscillatory they move until they lock at specific equilibrium distances. Typically at stable distances the intensity has a minimum in the middle point while it has a maximum for unstable equilibrium distances.

For oscillatory DS the scenario is richer. For typical system parameters ($I_s = 0.9$,$\theta = 1.27$) there are three equilibrium distances. If the initial separation is larger than $d = 19.8$ the oscillatory DS move until they reach the largest equilibrium distance $d_3 = 23.44$. The intensity has a minimum in the middle point and in fact this is a stable separation. Independently of the initial phase of the oscillatory DS, after a transient the two DS synchronize and oscillate in phase (See Fig.1 top row).

If the initial separation is smaller than 19.8 but larger than 11.63 then the oscillating DS move until their distance is $d_2 = 15.75$. As shown in the second row of Fig.1 at this distance the intensity in the middle point has in fact a maximum, so for static DS this would be an unstable equilibrium distance. For oscillatory DS this distance is stable and in fact it leads to an antiphase behavior.

Finally if the initial separation is smaller than 11.63 the two DS move to the smaller equilibrium distance $d = 7.69$. At this separation one encounters coexistence of in-phase (Fig.1 third row) and out-of-phase (Fig.1 bottom row) stable oscillations. The system evolves to one or the other depending on the initial condition. The period of in-phase oscillation ($T = 8.59$) is similar to that of a structure alone ($T = 8.66$) while the one of out-of-phase oscillations is significantly larger ($T = 10.45$).

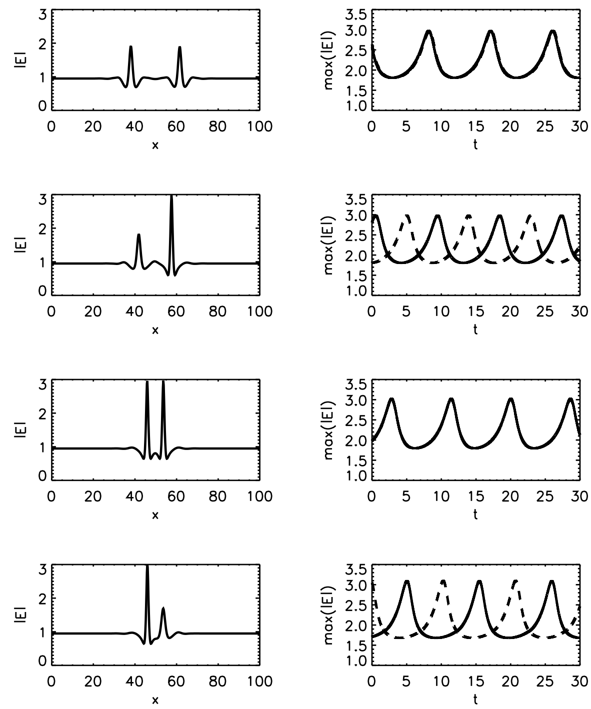

Figura 1. Left: Transverse profile of two oscillatory DS. Right: temporal evolution of the maximum of the left (solid) and right (dashed) structures.

* jacobo@ifisc.uib.es
† http://ifisc.uib.es

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