Language competition as an example of the consensus problem

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We consider an extension of the voter model in which a set of interacting elements (agents) can be in either of two equivalent states (A or B) or in a third additional mixed (AB) state. The model is motivated by studies of language competition dynamics, where the AB state is associated with bilingualism. Language competition belongs to the general class of processes that can be modelled by the interaction of heterogeneous agents as an example of collective phenomena in problems of social consensus[1]. We study the ordering process and associated interface and coarsening dynamics, addressing the role of the third AB state and the network of interactions.

Building upon a proposal by Minett and Wang[2], we study a model (*AB-model*) in which an agent i sits in a node within a network of N individuals and has k_i neighbors. It can be in three possible states: A, agent choosing option A (using language A); B, agent choosing option B (using language B); and AB, agent in a state of coexisting options (bilingual agent using both languages, A and B). States A and B are equivalent states.

The state of an agent evolves as follows: starting from a given initial condition, at each iteration we choose one agent i at random and we compute the local densities for each of the three communities in the neighborhood of node i, σ_i^l (l=A, B, AB). The agent changes its state according to the following transition probabilities[3]:

$$p_{i,A\to AB} = \frac{1}{2}\sigma_i^B, \qquad p_{i,B\to AB} = \frac{1}{2}\sigma_i^A \tag{1}$$

$$p_{i,AB\to B} = \frac{1}{2}(1-\sigma_i^A), \qquad p_{i,AB\to A} = \frac{1}{2}(1-\sigma_i^B). \tag{2}$$

For the voter model[4] the transition probabilities are simply given by $p_{i,A\to B} = \sigma_i^B$, $p_{i,B\to A} = \sigma_i^A$. The voter model rules are equivalent to the adoption by the agents of the opinion of a randomly chosen neighbor.

In a 2-dimensional regular lattice[5] we show that the typical size of a domain, $\langle\xi\rangle$, grows as $\langle\xi\rangle \sim t^{0.45}$. This result is compatible with the well known exponent 0.5 associated with domain growth driven by mean curvature and surface tension reduction observed in SFKI models. Agents in the AB state define the interfaces, changing the interfacial noise driven coarsening of the voter model to curvature driven coarsening. Moreover, around 1/3 of the realizations get trapped in long-lived metastable states. They correspond to stripe-like configurations for an A or B domain. For the realizations that do not get trapped in long lived metastable states, the characteristic time to reach an absorbing state can be estimated to scale as $\tau \sim N$. The global characteristic time instead, is dominated by the persistence of the dynamical metastable states, so that $\tau \sim N^\alpha$, with $\alpha \sim 1.8$.

In small world networks[5], AB agents restore coarsening, eliminating the metastable states of the voter model. The time to reach the absorbing state scales with system size as $\tau \sim \ln N$ to be compared with the result $\tau \sim N$ for the voter model in a small world network.

When taking into account a model network with community structure[6], the fraction of runs which have not reach consensus at time t decays exponentially in the voter model. In the AB-model instead, it appears to have power law behaviour $f(t) \sim t^{-\alpha}$, $\alpha \approx 1.3$. Since the exponent $\alpha < 2$, the average decay time for the bilinguals model does not give a characteristic time scale, but metastable states are found at any time scale (see Fig 1).

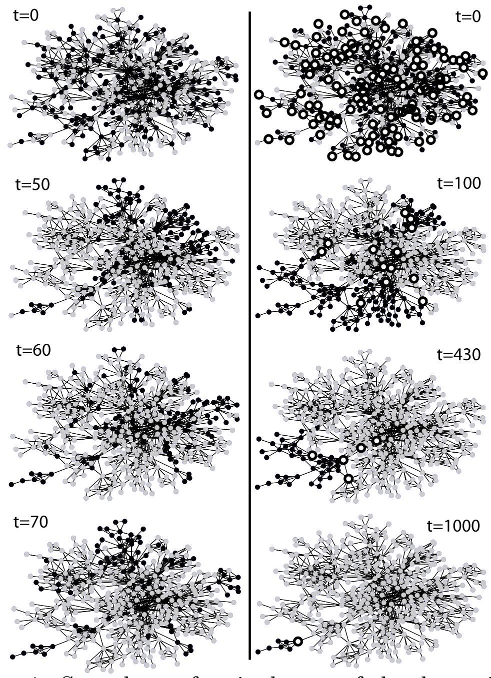

Figura 1. Snapshots of a single run of the dynamics, with nodes in state A in black, B in grey, and AB in white circled in black. Left: voter model. Right: AB-model (metastable states at $t = 430$ and $t = 1000$).

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[1] San Miguel M et al. 2005 Computing in Science and Engineering **7** Issue 6 67

[2] Wang W. S.-Y. and Minett J. W. 2005 Trends in Ecology and Evolution **20** (263)

[3] The prefactor 1/2 corresponds to equivalent A and B states.

[4] Holley R. and Liggett T. 1975 Ann. Probab.**4** (195)

[5] Castelló et al. 2006 New Journal of Physics **8** (308–322)

[6] Castelló et al. 2007 Europhysics Letters **79** (66006)