Oscillatory regime in excitatory media with global coupling. Application to cardiac dynamics.

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The study of the heart dynamics encompasses a broad range of scientific enquiry. They include from the study of the generation and propagation of the electric signal through the heart to the compression of the whole organ due to this propagation which gives rise to periodic blood pumping[1] and another whole range of different topics which should be understood to fully comprehend the different cardiac problems that might appear such as the control mechanisms that set-up the heart rhythm to the effect of the lack of oxygen in changing heart properties.

Probably one of the best investigated aspects of the heart dynamics is the electrical activity in the heart. The sinoatrial node, which is a collection of cells at the top of the atria, generates electrical impulses with a period mainly regulated by the sympathetic and the parasympathetic nervous system. This initial activity is propagated in the atrium, passes through the atrioventricular node, proceeds through the bundle of hiss to the Purkinje fibers reaching finally the whole ventricle. The dynamics of the currents involved at the level of the cell is well established and its macroscopic propagation can be accounted for obtaining experimentally the electric diffusion tensor at the different points of the heart[2].

Furthermore, the effects of the electric propagation in the elastic properties of the cell are well known. When the depolarization propagates through the cell one of the main currents entering it consists on calcium ions. Those turn on a set of complex mechanisms inside the cell which unblock the connection between the actin and myosin proteins present in the sarcomere of the cell: the bonding of those proteins generates internal forces which change the viscoelastic properties of the cell in the so-called active state[2]. Unfortunately, this knowledge at the level of the cell is very difficult to complete at the macroscopic level. There are different reasons for this difficulty. First, the passive properties of the heart as a viscoelastic material when no electrical activity is present are not generally known due to the high anisotropy of the tissue and the difficulty in performing experiments to measure tri-axial viscoelastic constants in reliable tissue. Second, it is even more difficult to asses the viscoelastic properties in the active state and relate them with cell properties given the combination of passive elements (collagen proteins, fibroblasts) and active elements (cardiomyocytes) which change in a highly history-dependent way.

To complicate the picture further, recent studies have shown a mechano-electric feedback where the stretching process of the heart affects the propagation of the electric signal due to gates in the cell membrane which are stretch-dependent. This is, the ability of ions to pass through these gates into the cell depends on the level of stretching in the cell membrane.

To face this complicated picture, different simplified models have been proposed to analyze what kind of dynamics and new behavior one may expect depending on the viscoelastic properties of the full heart and on the strength of the mechano-electric feedback. One typical example is the Panfilov model[3] which includes all the basic parts of the cardiac dynamics (except for the anisotropy) at the most basic level. It includes a two-variable model to explain the electrical propagation with a simple mechano-electric feedback. Another equation relates the voltage with the internal tension generated in the cell and, finally, an isotropic hypoelastic model is given to describe the mechanical properties of the tissue.

The purpose of this work is to show that as long as the stretching remains bounded at roughly 10 % and the boundaries of the system remain fixed, a model adequate to study the main effects of the feedback can be written in terms of an excitatory system with a global coupling. In particular, we will show that the model introduced by Panfilov can be written as:

$$\partial_t u = D\nabla^2 u - Ku(u-1)(u-a) - uz - I_{gc} \qquad (1)$$

$$I_{gc} = G(u-1)(\bar{z}-z)\Theta(\bar{z}-z) \quad \partial_t z = \epsilon(Ku - z) \qquad (2)$$

where u is the voltage, z is a slow variable related with the internal tension, and $\bar{z}$ is the instantaneous average value of V in the whole tissue giving rise to the global coupling I_{gc} (with D as the diffusion constant, G the strength of the global feedback and K, ϵ and a fixing the shape of the excitation front). This is basically a F-N model with a global coupling which has been investigated in other fields such as gas discharges or chemical reactions because it can help control the pattern formation process.

Finally we will apply this model to analyze one-dimensional cardiac tissue and study the different kind of patterns which have been found in the more complex mechanico-elastic models. We will study the evolution of the nullclines to explain how excitatory parts of the tissue can become oscillatory and when the mechano-electric feedback can generate spontaneous oscillatory regimes in otherwise excitatory media.

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[1] http://thevirtualheart.org/.

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