**Pattern formation and suppresion in periodic media**

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The formation of spatial structures and patterns in nonlinear optical cavities with added photonic crystals has attracted recent numerical and experimental interest. The unusual optical properties of structured materials allow for control of light in ways that are not possible with conventional optics. For instance, a photonic band-gap material can be used to inhibit the modulation instability leading to the formation of regular patterns [1].
Here we derive universal amplitude equations describing the formation of spatial structures in the presence of modulated media. We consider a prototypical model for pattern formation in nonlinear optical cavities: the singly resonant degenerate optical parametrical oscillator. In the mean-field approximation, the slowly varying amplitude of the signal field $A(x,t)$ is described by: $\partial_t A = -A - i\Delta - i\alpha f(x) + EA^* - |A|^2 A + i\partial_x^2 A$, where $\Delta$ is the average signal detuning, $f(x)$ describes the spatial modulation of the photonic crystal and $E$ is the amplitude of the external pump field. This simple system allows for analytical treatment in the context of the coupled-mode theory. We determine analytically the form of unstable modes and new delayed thresholds, corresponding to pattern suppression, via linear stability analysis. We show that, generically, the band-gap is divided in two separate regions: the upper one, in which pattern formation takes place due to an energy concentration at the minima of the photonic crystal modulation, and the lower one, in which energy concentrates at the maxima. We then determine amplitude equations for the unstable modes and find the stationary solutions above threshold analytically. We show that this scenario is organized by a codimension two bifurcation point where both modes become simultaneously unstable. We also show that, due to the broken translational invariance, there are always two different "phase locked" solutions. In broad area systems, this leads to the formation of domains of different "phases" connected by domain walls. This is a novel kind of symmetric bistability where the two "phases" do not differ by phase jumps of the electromagnetic field but by their relative position in the near field.

[1] D. Gomila et al. Phys. Rev. Lett. **92**, 253904 (2004).

[2] D. Gomila et al., Phys. Rev. E **72**, 016614 (2005).