CONVECTIVE INSTABILITY IN AN OPEN CHAIN OF SYNCHRONIZED IDENTICAL TIME-DELAY OSCILLATORS

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We study the stability properties of the synchronization manifold (SM) in an open chain of unidirectionally coupled identical time-delay oscillators. Despite being absolutely stable, the synchronization manifold is unstable to propagating perturbations. Convective instability has been also found in anticipating synchronization of chains of chaotic coupled oscillators (C. Mendoza et al., Phys. Rev. E**69**, 047202, 2004). We consider a chain of N coupled identical time-delay oscillators, given by $\dot{u}_i(t) = -au_i(t) + f(u_i(t-\tau)) + k_c(1-\delta_{1i})(u_{i-1}(t) - u_i(t))$ $(i = 1, \ldots N)$, where τ is the time delay and k_c is the coupling strength.
We focus the discussion of our numerical results on the Mackey-Glass model with $a = 0.1$ and $f(x) = 0.2x(t-\tau)/(1 + x(t-\tau)^{10})$, but we found similar results for the Ikeda system $(f_I(x) = \beta \sin^2(x + \phi))$. The Mackey-Glass system is chaotic for $\tau > 16.8$. Synchronization is achieved for large enough values of k_c. If the synchronized system is perturbed by changing x_1 by a small amount, the deviation from the SM initially grows but converges to 0 thus confirming its absolute stability. However, absolute stability of the SM is only a necessary condition for the robustness of synchronization. In fact it is found that the size of the deviation from the SM grows exponentially with i. Therefore the behavior of perturbations is analogous to that of convectively unstable systems: a localized perturbation appears to grow in suitably moving frames.
The asymptotic perturbation amplitude in site i at time t is $\delta(i, t = i/v) \sim \exp(\Lambda(v)t)$, where $\Lambda(v)$ is the convective Lyapunov exponent at the propagation velocity v. A positive maximum of $\Lambda(v)$, Λ_m, is found for a velocity v_m. The values of Λ_m and v_m are proportional to $1/\tau$ for long values of the delay time (chaotic regime). Then the amplification of the perturbation with i, given by $\delta(i, t = i/v) \sim \exp((\Lambda(v)/v)i) \sim \exp 0.2i$, is independent of τ. In the non-chaotic regime (low values of τ) the growth of the perturbation with i is reduced. In the chaotic regime it is found that Λ_m is independent of the coupling strength, whereas v_m is proportional to k_c.
Results on the effect of a delay in the coupling (anticipating synchronization: $k_c(u_{i-1}(t) - u_i(t - \tau_a))$) on the convective instability will be also reported.